

Fórmulas para o seno e cosseno da soma e da diferença entre dois arcos

$$\text{Sen}(a+b) = \text{Sen}(a) \cdot \text{Cos}(b) + \text{Sen}(b) \cdot \text{Cos}(a) \rightsquigarrow \text{Equação I}$$

$$\text{Sen}(a-b) = \text{Sen}(a) \cdot \text{Cos}(b) - \text{Sen}(b) \cdot \text{Cos}(a) \rightsquigarrow \text{Equação II}$$

$$\text{Cos}(a+b) = \text{Cos}(a) \cdot \text{Cos}(b) - \text{Sen}(a) \cdot \text{Sen}(b) \rightsquigarrow \text{Equação III}$$

$$\text{Cos}(a-b) = \text{Cos}(a) \cdot \text{Cos}(b) + \text{Sen}(a) \cdot \text{Sen}(b) \rightsquigarrow \text{Equação IV}$$

Equação I + Equação II:

$$\text{Sen}(a+b) + \text{Sen}(a-b) = \text{Sen}(a) \cdot \text{Cos}(b) + \text{Sen}(b) \cdot \text{Cos}(a) + \text{Sen}(a) \cdot \text{Cos}(b) - \text{Sen}(b) \cdot \text{Cos}(a)$$

$$\text{Sen}(a+b) + \text{Sen}(a-b) = 2 \cdot \text{Sen}(a) \cdot \text{Cos}(b)$$

$$\boxed{\text{Sen}(a) \cdot \text{Cos}(b) = \frac{1}{2} \cdot (\text{Sen}(a+b) + \text{Sen}(a-b))} \quad \text{Fórmula de Werner}$$

Seja $a+b=p$ e $a-b=q$

$$\begin{array}{r} a+b=p \\ + a-b=q \\ \hline 2a=p+q \Rightarrow a=\frac{p+q}{2} \end{array} \quad \begin{array}{r} a+b=p \\ - a-b=q \\ \hline 2b=p-q \Rightarrow b=\frac{p-q}{2} \end{array}$$

$$\text{Sen}\left(\frac{p+q}{2}\right) \cdot \text{Cos}\left(\frac{p-q}{2}\right) = \frac{1}{2} \cdot (\text{Sen}(p) + \text{Sen}(q))$$

$$\boxed{\text{Sen}(p) + \text{Sen}(q) = 2 \cdot \text{Sen}\left(\frac{p+q}{2}\right) \cdot \text{Cos}\left(\frac{p-q}{2}\right)} \quad \text{Fórmula de Prostaferese}$$

Equação I - Equação II:

$$\text{Sen}(a+b) - \text{Sen}(a-b) = \text{Sen}(a) \cdot \text{Cos}(b) + \text{Sen}(b) \cdot \text{Cos}(a) - \text{Sen}(a) \cdot \text{Cos}(b) + \text{Sen}(b) \cdot \text{Cos}(a)$$

$$\text{Sen}(a+b) - \text{Sen}(a-b) = 2 \cdot \text{Sen}(b) \cdot \text{Cos}(a)$$

$$\boxed{\text{Sen}(b) \cdot \text{Cos}(a) = \frac{1}{2} \cdot (\text{Sen}(a+b) - \text{Sen}(a-b))} \quad \text{Fórmula de Werner}$$

Seja $a+b=p$ e $a-b=q$

$$\begin{array}{r} a+b=p \\ + a-b=q \\ \hline 2a=p+q \Rightarrow a=\frac{p+q}{2} \end{array} \quad \begin{array}{r} a+b=p \\ - a-b=q \\ \hline 2b=p-q \Rightarrow b=\frac{p-q}{2} \end{array}$$

$$\text{Sen} \left(\frac{p-q}{2} \right) \cdot \text{Cos} \left(\frac{p+q}{2} \right) = \frac{1}{2} \cdot (\text{Sen}(p) - \text{Sen}(q))$$

$$\boxed{\text{Sen}(p) - \text{Sen}(q) = 2 \cdot \text{Sen} \left(\frac{p-q}{2} \right) \cdot \text{Cos} \left(\frac{p+q}{2} \right)} \quad \text{Fórmula de Prostaferese}$$

Equação III + Equação IV:

$$\text{Cos}(a+b) + \text{Cos}(a-b) = \text{Cos}(a) \cdot \text{Cos}(b) - \text{Sen}(a) \cdot \text{Sen}(b) + \text{Cos}(a) \cdot \text{Cos}(b) + \text{Sen}(a) \cdot \text{Sen}(b)$$

$$\text{Cos}(a+b) + \text{Cos}(a-b) = 2 \cdot \text{Cos}(a) \cdot \text{Cos}(b)$$

$$\boxed{\text{Cos}(a) \cdot \text{Cos}(b) = \frac{1}{2} \cdot (\text{Cos}(a+b) + \text{Cos}(a-b))} \quad \text{Fórmula de Werner}$$

Seja $a+b=p$ e $a-b=q$

$$\begin{array}{r} a+b=p \\ + a-b=q \\ \hline 2a=p+q \Rightarrow a=\frac{p+q}{2} \end{array} \quad \begin{array}{r} a+b=p \\ - a-b=q \\ \hline 2b=p-q \Rightarrow b=\frac{p-q}{2} \end{array}$$

$$\text{Cos} \left(\frac{p+q}{2} \right) \cdot \text{Cos} \left(\frac{p-q}{2} \right) = \frac{1}{2} \cdot (\text{Cos}(p) + \text{Cos}(q))$$

$$\boxed{\text{Cos}(p) + \text{Cos}(q) = 2 \cdot \text{Cos} \left(\frac{p+q}{2} \right) \cdot \text{Cos} \left(\frac{p-q}{2} \right)} \quad \text{Fórmula de Prostaferese}$$

Equação III - Equação IV:

$$\text{Cos}(a+b) - \text{Cos}(a-b) = \text{Cos}(a) \cdot \text{Cos}(b) - \text{Sen}(a) \cdot \text{Sen}(b) - \text{Cos}(a) \cdot \text{Cos}(b) - \text{Sen}(a) \cdot \text{Sen}(b)$$

$$\text{Cos}(a+b) - \text{Cos}(a-b) = -2 \cdot \text{Sen}(a) \cdot \text{Sen}(b)$$

$$\boxed{\text{Sen}(a) \cdot \text{Sen}(b) = -\frac{1}{2} \cdot (\text{Cos}(a+b) - \text{Cos}(a-b))} \quad \text{Fórmula de Werner}$$

Seja $a+b=p$ e $a-b=q$

$$\begin{array}{r} a+b=p \\ + a-b=q \\ \hline 2a=p+q \Rightarrow a=\frac{p+q}{2} \end{array} \quad \begin{array}{r} a+b=p \\ - a-b=q \\ \hline 2b=p-q \Rightarrow b=\frac{p-q}{2} \end{array}$$

$$\text{Sen} \left(\frac{p+q}{2} \right) \cdot \text{Sen} \left(\frac{p-q}{2} \right) = -\frac{1}{2} \cdot (\text{Cos}(p) - \text{Cos}(q))$$

$$\boxed{\text{Cos}(p) - \text{Cos}(q) = -2 \cdot \text{Sen} \left(\frac{p+q}{2} \right) \cdot \text{Sen} \left(\frac{p-q}{2} \right)} \quad \text{Fórmula de Prostaferese}$$

$$\text{Sen}(a \pm b) = \text{Sen}(a) \cdot \text{Cos}(b) \pm \text{Sen}(b) \cdot \text{Cos}(a)$$

$$\text{Cos}(a \pm b) = \text{Cos}(a) \cdot \text{Cos}(b) \mp \text{Sen}(a) \cdot \text{Sen}(b)$$

Fórmulas de Werner

transformam o produto em soma

$$\text{Sen}(a) \cdot \text{Cos}(b) = \frac{1}{2} \cdot (\text{Sen}(a+b) + \text{Sen}(a-b))$$

$$\text{Cos}(a) \cdot \text{Sen}(b) = \frac{1}{2} \cdot (\text{Sen}(a+b) - \text{Sen}(a-b))$$

$$\text{Cos}(a) \cdot \text{Cos}(b) = \frac{1}{2} \cdot (\text{Cos}(a-b) + \text{Cos}(a+b))$$

$$\text{Sen}(a) \cdot \text{Sen}(b) = \frac{1}{2} \cdot (\text{Cos}(a-b) - \text{Cos}(a+b))$$

Fatoração Trigonométrica (Fórmulas de Prostaferese)

transformam a soma em produto

$$\text{Sen}(p) \pm \text{Sen}(q) = 2 \cdot \text{Sen}\left(\frac{p \pm q}{2}\right) \cdot \text{Cos}\left(\frac{p \mp q}{2}\right)$$

$$\text{Cos}(p) + \text{Cos}(q) = 2 \cdot \text{Cos}\left(\frac{p+q}{2}\right) \cdot \text{Cos}\left(\frac{p-q}{2}\right)$$

$$\text{Cos}(p) - \text{Cos}(q) = -2 \cdot \text{Sen}\left(\frac{p+q}{2}\right) \cdot \text{Sen}\left(\frac{p-q}{2}\right)$$

$$\text{tg}(p) \pm \text{tg}(q) = \frac{\text{Sen}(p \pm q)}{\text{Cos}(p) \cdot \text{Cos}(q)}$$